Supplementary information

April 16, 2008

Development of collective modes

The atoms in our system are confined at many locations within a one dimensional optical lattice of wavevector $k_t = 2\pi/850$ nm, yet interact with the cavity mode at the position dependent coupling rate $g(z) = g_0 \sin k_p z$. Because of the position dependent coupling and the distribution of the atoms, a single collective degree of freedom (but not a center of mass degree of freedom) interacts with the cavity mode.

We consider that each atom is trapped harmonically with frequency ω_z and trap center $\bar{z}_i$, where we denote the displacement of atom i from its trap's center by the operator $\delta z_i = z_i - \bar{z}_i = z_{\rm ho}(\hat{a}_i^\dagger + \hat{a}_i)$ with $z_{\rm ho} = \sqrt{\hbar/2m\omega_z}$ being the harmonic oscillator length and atom field operators $\hat{a}_i$ and $\hat{a}_i^\dagger$ conventionally defined. We assume that the atomic displacements are small ($k_p \delta z_i \ll 1$) and that the cavity-atom detuning is large ($|\Delta_{ca}| \gg g_0\sqrt{N}$). Omitting some constant terms, we obtain a Hamiltonian describing the coupled atoms/cavity system as

$$\mathcal{H} = \left(\hbar\omega_c + \sum_i \left[\frac{\hbar g^2(\bar{z}_i)}{\Delta_{ca}} - f_i \delta z_i\right]\right) n + \mathcal{H}_a + \mathcal{H}_{in}, \tag{1}$$

where n is the cavity photon number operator, $\mathcal{H}_a = \sum_i \hbar\omega_z \hat{a}_i^\dagger \hat{a}_i$, and $\mathcal{H}_{in}$ describes optical modes external to the cavity and their coupling to the cavity field [1]. Here, the per-atom cavity resonance shift is expanded to first order in the atomic position operator, with $f_i = -\hbar\partial_z g^2(z)/\Delta_{ca} = f_0 \sin(2k_p\bar{z}_i)$ being the optical dipole force on atom i from a single cavity photon.

We define a collective position operator $Z = (N_{\rm eff})^{-1}\sum_i \sin(2k_p\bar{z}_i)\delta z_i$, and the conjugate momentum $P = \sum_i \sin(2k_p\bar{z}_i) p_i$, with p_i being the momentum of atom i, and $N_{\rm eff} = \sum_i \sin^2(2k_p\bar{z}_i)$. The cavity then serves to monitor a specific collective mode of motion in the atomic ensemble, with the cavity resonance being shifted by $\Delta_N - N_{\rm eff}\; f_0 Z/\hbar$ where $\Delta_N = \sum_i g(\bar{z}_i)^2/\Delta_{ca}$ is the cavity frequency shift with all atoms localized at their potential minima.

Given these collective operators we can write equations of motion;

$$\dot{P} = \sum_i \sin(2k_p\bar{z}_i)\Big(-m\omega_z^2 \delta z_i + f_i n\Big) \tag{2}$$

$$= -N_{\rm eff}\; m\omega_z^2\Big(Z - \frac{f_0 \bar{n}}{m\omega_z^2}\Big) + f_0(n - \bar{n}). \tag{3}$$

$$\dot{Z} = \frac{P}{N_{\rm eff}\; m}. \tag{4}$$

A constant average optical force of $\bar{n}$ cavity photons displaces the collective position variable by $\Delta Z = (\hbar k g_0^2/m\omega_z^2\Delta_{ca})\bar{n}$ and thereby shifts the cavity resonance frequency to $\omega_c' = \omega_c + \Delta_N - N_{\text{eff}} f_0 \Delta Z/\hbar$. We define collective quantum operators a and $a^\dagger$ through the relations $Z - \Delta Z = Z_{\text{ho}}(a^\dagger + a)$ and $P = iP_{\text{ho}}(a^\dagger - a)$, with $Z_{\text{ho}} = z_{\text{ho}}/\sqrt{N_{\text{eff}}}$ and $P_{\text{ho}} = \hbar/(2Z_{\text{ho}})$. With these substitutions, we have the Hamiltonian describing the collective mode–cavity system:

$$\mathcal{H} = \hbar\omega_c' n - N_{\text{eff}} f_0 Z_{\text{ho}}(a^\dagger + a)(n - \bar{n}) + \hbar\omega_z a^\dagger a + \mathcal{H}_{in}. \tag{5}$$

Calculation of the heating rate

Given equation 5, we can draw directly on existing results which analyze cavity cooling and heating for similar Hamiltonians [2]. For clarity, however, we present a derivation of the heating rate below. From Eq. 5 we obtain equations of motion for a and for the cavity field operator b.

$$\frac{da}{dt} = -i\omega_z a + i\kappa\epsilon(n - \bar{n}), \tag{6}$$

$$\frac{db}{dt} = -i\omega_c' b + i\kappa\epsilon(a^\dagger + a)b - \kappa b + \sqrt{2\kappa} b_{in}, \tag{7}$$

where κ is the decay rate of the cavity field and b_{in} represents the coherent-state input field that drives the cavity. We have introduced the granularity parameter $\epsilon = N_{\text{eff}} f_0 Z_{\text{ho}}/(\hbar\kappa)$ as discussed in the text. We can now express the atomic field operator as,

$$a(t) = e^{-i\omega_z t} a(0) + i\kappa\epsilon \int_0^t dt' e^{-i\omega_z(t-t')}(n(t') - \bar{n}). \tag{8}$$

From here, we evaluate the rate of change of the atomic energy:

$$\frac{d}{dt}(a^\dagger a) = \Big(\frac{d}{dt}a^\dagger\Big)_t a(t) + a^\dagger(t)\Big(\frac{d}{dt}a\Big)_t \tag{9}$$

$$= \Big[i\omega_z a^\dagger(t) - i\kappa\epsilon(n(t) - \bar{n})\Big]a(t) + a^\dagger(t)\Big[-\omega_z a^\dagger(t) + i\kappa\epsilon(n(t) - \bar{n})\Big] \tag{10}$$

$$= 2\kappa^2\epsilon^2 \ \mathrm{Re}\Bigg[\int_0^t dt'(n(t) - \bar{n})(n(t') - \bar{n})e^{-i\omega_z(t-t')}\Bigg] + i\kappa\epsilon\Big(a^\dagger(0)(n(t) - \bar{n})e^{i\omega_z t} - (n(t) - \bar{n})a(0)e^{-i\omega_z t}\Big). \tag{11}$$

For the sake of evaluating the cavity field evolution we restrict our treatment to times which are short compared to the timescale over which the atomic motion is significantly varied by interaction with the light. Under this ansatz we approximate Eq. 8 as

$$a(t) \simeq e^{-i\omega_z t} a(0). \tag{12}$$

Inserting this solution for the atomic field operator into the equation of motion for the cavity field, (7) we have the following for the frequency components of b:

$$-i\omega b(\omega) = -i\omega_c' b(\omega) - \kappa b(\omega) + \sqrt{2\kappa} b_{in}(\omega) + i\kappa\epsilon\big(a(0)b(\omega - \omega_z) + a^\dagger(0)b(\omega + \omega_z)\big). \tag{13}$$

Defining $L(\omega) = (1 - i(\omega - \omega_c')/\kappa)^{-1}$, we obtain

$$b(\omega) = \frac{L(\omega)}{\kappa}\Big[\sqrt{2\kappa}b_{in}(\omega) + i\epsilon\Big(a(0)b(\omega-\omega_z) + a^\dagger(0)b(\omega+\omega_z)\Big)\Big]. \tag{14}$$

We can solve this equation iteratively,

$$\begin{aligned} b(\omega) = \frac{L(\omega)}{\kappa}\Big[\sqrt{2\kappa}b_{in}(\omega) + i\epsilon\sqrt{2\kappa}\Big(a(0)L(\omega-\omega_z)b_{in}(\omega-\omega_z) \\ +a^\dagger(0)L(\omega+\omega_z)b_{in}(\omega+\omega_z)\Big) + \mathcal{O}(|\epsilon a(0)|^2)\Big]. \end{aligned} \tag{15}$$

In the non-granular regime $\epsilon \ll 1$, and assuming small values of $a(0)$, i.e. that the atoms are sufficiently cold, we neglect terms of order ϵ^3 or higher.

Returning to Eq. 11 we now have

$$n(t) = \frac{1}{2\pi}\int d\omega_1\ d\omega_2 e^{i(\omega_1-\omega_2)t}\ b^\dagger(\omega_1)b(\omega_2) \tag{16}$$

$$\begin{aligned} &= \frac{1}{2\pi}\int d\omega_1\ d\omega_2 e^{i(\omega_1-\omega_2)t}\frac{L^*(\omega_1)L(\omega_2)}{\kappa^2}2\kappa\Bigg[b_{in}^\dagger(\omega_1)b_{in}(\omega_2)+ \\ &i\epsilon b_{in}^\dagger(\omega_1)\Big(a(0)L(\omega_2-\omega_z)b_{in}(\omega_2-\omega_z) + a^\dagger(0)L(\omega_2+\omega_z)b_{in}(\omega_2+\omega_z)\Big)- \\ &i\epsilon\Big(a^\dagger(0)L^*(\omega_1-\omega_z)b_{in}^\dagger(\omega_1-\omega_z) + a(0)L^*(\omega_1+\omega_z)b_{in}^\dagger(\omega_1+\omega_z)\Big)b_{in}(\omega_2)\Bigg]. \end{aligned} \tag{17}$$

With the above normally ordered product of operators b_{in} we are justified in replacing:

$$b_{in}(\omega) \to \sqrt{\pi\kappa n_{\max}}\,\delta(\omega-\omega_p), \tag{18}$$

$$b_{in}^\dagger(\omega) \to \sqrt{\pi\kappa n_{\max}}\,\delta(\omega-\omega_p), \tag{19}$$

where ω_p is the frequency of a probe laser, and $n_{\max}$ is the maximum intracavity photon number for resonant cavity excitation. Finally, we obtain

$$\begin{aligned} n(t) =& \bar{n}\Big[1 + i\epsilon\Big(a(0)L(\omega_p+\omega_z)e^{-i\omega_z t} + a^\dagger(0)L(\omega_p-\omega_z)e^{+i\omega_z t}\Big)- \\ & i\epsilon\Big(a^\dagger(0)L^*(\omega_p+\omega_z)e^{+i\omega_z t} + a(0)L^*(\omega_p-\omega_z)e^{-i\omega_z t}\Big)\Big]. \end{aligned} \tag{20}$$

Here we have substituted $\bar{n} = n_{\max}|L(\omega_p)|^2$.

We are now in a position to evaluate the heating rate:

$$\frac{d}{dt}E = \hbar\omega_z\Big\langle\frac{d}{dt}a^\dagger a\Big\rangle \tag{21}$$

$$\begin{aligned} &= 2\hbar\omega_z\kappa^2\epsilon^2\mathrm{Re}\Bigg[\int_0^t dt'\langle(n(t)-\bar{n})(n(t')-\bar{n})\rangle e^{-i\omega_z(t-t')}\Bigg]+ \\ &i\hbar\omega_z\kappa\epsilon\Big\langle a^\dagger(0)(n(t)-\bar{n})e^{i\omega_z t} - (n(t)-\bar{n})a(0)e^{-i\omega_z t}\Big\rangle. \end{aligned} \tag{22}$$

Addressing the first term first; for a linear cavity driven by a constant coherent state input, we substitute the relation,

$$\langle n(\tau)n(0)\rangle - \langle n(\tau)\rangle^2 = \bar{n}e^{i(\omega_p-\omega_c')\tau-\kappa\tau}. \tag{23}$$

Assuming the system is in a steady state, in that $\langle n(t)n(t')\rangle = \langle n(t-t')n(0)\rangle$, and substituting $\bar{n}^2 = \langle n(\tau)\rangle^2$ we obtain for the first half of the heating rate,

$$2\hbar\omega_z\kappa\epsilon^2\bar{n}\Big(\frac{1}{1+(\omega_p-\omega_c'-\omega_z)^2/\kappa^2}\Big) = \hbar\omega_z\kappa^2\epsilon^2[S_{nn}^{(-)}(\omega_z)]. \tag{24}$$

Here we have introduced the spectral density of photon number fluctuations $S_{nn}^{(\pm)}(\omega) = 2\bar{n}\kappa(\kappa^2+(\Delta\pm\omega)^2)^{-1}$ [2], with $\Delta = \omega_p - \omega_c'$ begin the probe detuning from the atoms shifted cavity resonance. The backaction heating is enhanced near the peak of the cavity resonance, where the sensitivity of the cavity field to the atomic displacement is strongest. We note that this heating rate can also be derived on a single atom basis, assuming the cavity mediated coupling between atoms is small.

The second term in Eq. 22 accounts for the effect of transient atomic motion on the cavity field. To evaluate this term we take the time average over an atomic oscillation period.

$$i\kappa\epsilon\Big\langle a^\dagger(0)(n(t)-\bar{n})e^{i\omega_z t} - (n(t)-\bar{n})a(0)e^{-i\omega_z t}\Big\rangle \tag{25}$$

$$= \bar{n}\epsilon^2\kappa\Big(L(\omega_p+\omega_z) - L^*(\omega_p-\omega_z) + L(\omega_p-\omega_z) - L^*(\omega_p+\omega_z)\Big)\langle a^\dagger(0)a(0)\rangle \tag{26}$$

$$= \kappa^2\epsilon^2\big[S_{nn}^{(-)}(\omega_z) - S_{nn}^{(+)}(\omega_z)\big]\langle a^\dagger a\rangle. \tag{27}$$

These terms represent cavity cooling/anti-cooling. In total, the change in energy is,

$$\frac{d}{dt}E = \hbar\omega_z\kappa^2\epsilon^2\left[S_{nn}^{(-)}(\omega_z) + \Big(S_{nn}^{(-)}(\omega_z) - S_{nn}^{(+)}(\omega_z)\Big)\langle a^\dagger a\rangle\right]. \tag{28}$$

Measuring backaction heating by the evaporative loss of trapped atoms

The accuracy of our measurement depends on assumptions made in interpreting the observed transmission lineshapes, several of which we verified experimentally. For example, we examined the dynamics of evaporative cooling in the atomic medium. For this, we interrupted the cavity transmission measurement, released the atoms from the intracavity optical trap and imaged them 4 ms later to measure their temperature. Within our measurement resolution of 0.1 μK, this temperature remained constant. Thus, our quantification of heating through the rate of atom loss is valid. Furthermore, by extinguishing the cavity probe light momentarily during cavity probing, and comparing the cavity transmission when the probe was turned off and then turned on again, we determined a timescale of 3 ms for N to equilibrate by evaporative cooling following an increase of thermal energy of the collective mode. Since this timescale is short compared to the $\simeq$ 100 ms span of the resonant transmission signal, we are justified in using simultaneous measurements of dN/dt and $\bar{n}$ to determine the instantaneous heating rate.

To interpret our measurements as relating to the quantum nature of the intracavity field, it was necessary to establish that quantum fluctuations dominate over classical, technical intensity fluctuations which would also lead to heating [3]. For this, we measured the light-induced heating for varying probe intensities, with $\bar{n}$ at the cavity resonance ranging from $\bar{n} = 0.2$ to 20. Noting that the contribution of quantum fluctuations to the atom heating rate scales as $\bar{n}$ while that of technical fluctuations scales as $\bar{n}^2$, we find that technical fluctuations account for less than 10% of the atom heating rate at the light level used for Figs. 2 and 3.

Visibility of photon fluctuations outside the cavity

In this section, we provide support for the observation that the spectrum of intracavity quantum fluctuations of the photon number is not visible in light transmitted through the coherently driven cavity. For a two sided cavity we have [1]:

$$c_{out}(t) + c_{in}(t) = \sqrt{\kappa} b(t), \tag{29}$$

$$d_{out}(t) + d_{in}(t) = \sqrt{\kappa} b(t), \tag{30}$$

$$b(\omega) = \frac{\sqrt{\kappa} c_{in}(\omega) + \sqrt{\kappa} d_{in}(\omega)}{\kappa - i(\omega - \omega_c')}. \tag{31}$$

The operators $c_{out}, d_{out}, c_{in}, d_{in}$ are photon annihilation operators for the outgoing and ingoing fields on either side of the cavity, and b is again the cavity field annihilation operator. The known commutation relations are,

$$\left[c_{in}(\omega_1), c_{in}^\dagger(\omega_2)\right] = \delta(\omega_1 - \omega_2), \qquad \left[c_{in}(t_1), c_{in}^\dagger(t_2)\right] = \delta(t_1 - t_2), \tag{32}$$

and similarly for d_{in}. To examine the spectrum of photon fluctuations inside the cavity we calculate the commutation relation for the cavity field operator:

$$\left[b(t_1), b^\dagger(t_2)\right] = \int \frac{d\omega_1\, d\omega_2}{2\pi} \frac{e^{-i\omega_1 t_1}}{\kappa - i(\omega_1 - \omega_c')} \frac{e^{i\omega_2 t_2}}{\kappa + i(\omega_2 - \omega_c')} \times \left[\sqrt{\kappa} c_{in}(\omega_1) + \sqrt{\kappa} d_{in}(\omega_1), \sqrt{\kappa} c_{in}^\dagger(\omega_2) + \sqrt{\kappa} d_{in}^\dagger(\omega_2)\right] \tag{33}$$

$$= \int \frac{d\omega_1\, d\omega_2}{2\pi} \frac{e^{-i\omega_1 t_1}}{\kappa - i(\omega_1 - \omega_c')} \frac{e^{i\omega_2 t_2}}{\kappa + i(\omega_2 - \omega_c')} 2\kappa \delta(\omega_1 - \omega_2) \tag{34}$$

$$= \int \frac{d\omega}{2\pi} \frac{2\kappa}{\kappa^2 + (\omega - \omega_c')^2} e^{i\omega(t_2 - t_1)} \tag{35}$$

$$= e^{i\omega_c'(t_2 - t_1) - \kappa |t_2 - t_1|}. \tag{36}$$

From this we obtain the two-time correlation in Eq. 23. Now, for the cavity output, (say,

d_{out}),

$$\left[d_{out}(t_1), d^\dagger_{out}(t_2)\right] = \int \frac{d\omega_1\ d\omega_2}{2\pi} e^{i\omega_1 t_1 + i\omega_2 t_2} \left[\frac{\kappa c_{in}(\omega_1) + \kappa d_{in}(\omega_1)}{\kappa - i(\omega_1 - \omega'_c)} - d_{in}(\omega_1), \right.$$

$$\left. \frac{\kappa c^\dagger_{in}(\omega_2) + \kappa d^\dagger_{in}(\omega_2)}{\kappa + i(\omega_2 - \omega'_c)} - d^\dagger_{in}(\omega_2)\right] \quad (37)$$

$$= \int \frac{d\omega_1\ d\omega_2}{2\pi} e^{i\omega_1 t_1 + i\omega_2 t_2} \left(\frac{2\kappa^2 \delta(\omega_1 - \omega_2)}{(\kappa - i(\omega_1 - \omega'_c))(\kappa + i(\omega_2 - \omega'_c))} - \right.$$

$$\left. \frac{\kappa \delta(\omega_1 - \omega_2)}{\kappa - i(\omega_1 - \omega'_c)} - \frac{\kappa \delta(\omega_1 - \omega_2)}{\kappa + i(\omega_2 - \omega'_c)}\right) \quad (38)$$

$$= \int \frac{d\omega}{2\pi} e^{i\omega(t_2 - t_1)} = \delta(t_1 - t_2). \quad (39)$$

The commutation relations for fields outside the cavity are the same as for light entering the cavity, and do not carry any evidence of the photon number dynamics (Eq. 36) inside the cavity.

*References

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[2] Marquardt, F. et al. Quantum theory of cavity-assisted sideband cooling of mechanical motion. *Phys. Rev. Lett.* **99**, 093902 (2007).

[3] Gehm, M. et al. Dynamics of noise-induced heating in atom traps. *Phys. Rev. A* **58**, 3914 (1998).

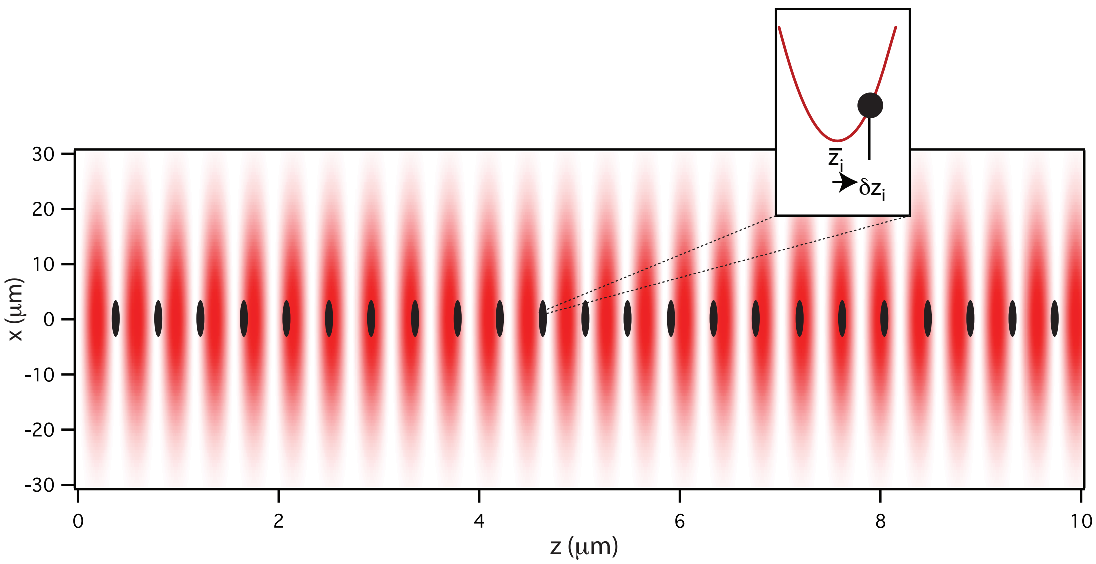

Figure 1: Schematic of the intracavity optical lattice. The $1/e$ width of the atomic distribution (back) is small compared to variations in the probe intensity (red linear density plot). At each location, harmonic confinement is provided by the lattice at 850 nm, and the atomic distribution occupies the ground state of the ω_z confinement. Each atom's position is given by the location of the minimum of the harmonic confinement, $\bar{z}_i$ plus a deviation from that minimum, δz_i.